Operators and Matrices
Volume 6, Number 3 (2012), 535–542
doi:10.7153/oam-06-36

POSITIVE COMMUTATORS AND COLLECTIONS OF OPERATORS

ROMAN DRNOVŠEK, MARTIN JESENKO AND MARKO KANDIĆ

Abstract. Let A and B be completely decomposable nonnegative matrices such that the commutator $AB-BA$ is also a nonnegative matrix. We prove that the set $\{A,B\}$ is completely decomposable, i.e., there exists a permutation matrix P such that PAP^{-1} and PBP^{-1} are upper triangular matrices. We show similar results for collections of completely decomposable nonnegative matrices. We also find conditions on commutators under which a given operator on a Riesz space is necessarily scalar.

Mathematics subject classification (2010): 15A48, 20M20.
Keywords and phrases: Nonnegative matrices, semigroups of matrices, indecomposability, commutators.

REFERENCES

[1] C. D. ALIPRANTIS, O. BURKINSHAW, *Positive operators*, Springer, 2006.
[2] J. BRAČIČ, R. DRNOVŠEK, Y. B. FARFOROVSKAYA, E. L. RABKIN, J. ZEMÁNEK, *On positive commutators*, Positivity **14**, 3 (2010), 431–439, DOI: 10.1007/s11117-009-0028-1.
[3] R. DRNOVŠEK, M. KANDIĆ, *Ideal-triangularizability of semigroups of positive operators*, Integral Equations Operator Theory **64**, 4 (2009), 539–552.
[4] H. RADJAVI, P. ROSENTHAL, *Simultaneous Triangularization*, Springer-Verlag, New York, 2000.

Paper OaM-06-36