

## ELEMENTARY MODELS OF UNBOUNDED JACOBI MATRICES WITH A FEW BOUNDED GAPS IN THE ESSENTIAL SPECTRUM

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*Abstract.* This work contains a constructive example of a class of Jacobi operators with an arbitrary finite number of gaps in its unbounded essential spectrum. The construction of this class is based on elementary ideas of gluing finite-dimensional Jacobi matrices whose sizes grow to infinity. The precise analysis of the finite-dimensional pieces leads to a new “finite essential spectrum” besides the natural essential spectrum of two explicit infinite Jacobi matrices, determined by the above finite dimensional ones. This new finite essential spectrum is calculated explicitly. A connection to the ideas of the recent paper [12] is also given.

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