THE POSSIBLE SHAPES OF NUMERICAL RANGES

J. WILLIAM HELTON AND I. M. SPITKOVSKY

Abstract. Which convex subsets of $\mathbb{C}$ are the numerical range $W(A)$ of some matrix A? This paper gives a precise characterization of these sets. In addition to this we show that for any A there exists a symmetric B of the same size such that $W(A) = W(B)$ thereby settling an open question from [2].

REFERENCES

[1] A. BEN-TAL AND A. NEMIROVSKI, *Lectures on modern convex optimization*, MPS/SIAM Series on Optimization, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2001, Analysis, algorithms, and engineering applications.

[2] W.-S. CHEUNG, X. LIU, AND T.-Y. TAM, *Multiplicities, boundary points, and joint numerical ranges*, Operators and Matrices **5**, 1 (2011), 41–52.

[3] K. E. GUSTAFSON AND D. K. M. RAO, *Numerical range. The field of values of linear operators and matrices*, Springer, New York, 1997.

[4] J. W. HELTON AND V. VINNIKOV, *Linear matrix inequality representation of sets*, Comm. Pure Appl. Math. **60**, 5 (2007), 654–674.

[5] D. HENRION, *Semidefinite geometry of the numerical range*, Electron. J. Linear Algebra **20** (2010), 322–332.

[6] R. A. HORN AND C. R. JOHNSON, *Topics in matrix analysis*, Cambridge University Press, Cambridge, 1991.

[7] R. KIPPENHAHN, *Über den Wertevorrat einer Matrix*, Math. Nachr. **6** (1951), 193–228.

[8] R. KIPPENHAHN, *On the numerical range of a matrix*, Linear Multilinear Algebra **56**, 1–2 (2008), 185–225; Translated from the German by Paul F. Zachlin and Michiel E. Hochstenbach [MR0059242].

[9] R. T. ROCKAFELLAR, *Convex analysis*, Princeton Landmarks in Mathematics, Princeton University Press, Princeton, NJ, 1997, Reprint of the 1970 original, Princeton Paperbacks.

[10] P. ROSTLASKI AND B. STURMFELS, *Dualities in convex algebraic geometry*, Rendiconti di Matematica, Serie VII **30** (2010), 285–327.

© ELEMENT, Zagreb
Paper OaM-06-41

Operators and Matrices
www.ele-math.com
oam@ele-math.com