

ORTHOGONALLY ADDITIVE HOLOMORPHIC FUNCTIONS ON C^* -ALGEBRAS

ANTONIO M. PERALTA AND DANIELE PUGLISI

Abstract. Let A be a C^* -algebra. We prove that a holomorphic function of bounded type $f : A \rightarrow \mathbb{C}$ is orthogonally additive on A_{sa} if, and only if, it is additive on elements having zero-product if, and only if, there exist a positive functional φ in A^* , a sequence (ψ_n) in $L_1(A^{**}, \varphi)$ and a power series holomorphic function h in $\mathcal{H}_b(A, A^*)$ such that

$$h(a) = \sum_{k=1}^{\infty} \psi_k \cdot a^k \text{ and } f(a) = \langle 1_{A^{**}}, h(a) \rangle = \int h(a) d\varphi,$$

for every a in A , where $1_{A^{**}}$ denotes the unit element in A^{**} and $L_1(A^{**}, \varphi)$ is a non-commutative L_1 -space.

Mathematics subject classification (2010): Primary 46G20, 46L05; Secondary 46L51, 46E15, 46E50.

Keywords and phrases: C^* -algebra, von Neumann algebra, orthogonally additive holomorphic functions, non-commutative L_1 -spaces.

REFERENCES

- [1] Y. BENYAMINI, S. LASSALLE, AND J. G. LLAVONA, *Homogeneous orthogonally additive polynomials on Banach lattices*, Bull. London Math. Soc. **38**, 3 (2006), 459–469.
- [2] M. BURGOS, F. J. FERNÁNDEZ-POLO, J. J. GARCÉ, AND A. M. PERALTA, *Orthogonality preservers Revisited*, Asian-European Journal of Mathematics **2**, 3 (2009), 387–405.
- [3] D. CARANDO, S. LASSALLE, AND I. ZALDUENDO, *Orthogonally additive polynomials over $C(K)$ are measures – a short proof*, Integr. equ. oper. theory **56** (2006), 597–602.
- [4] D. CARANDO, S. LASSALLE, AND I. ZALDUENDO, *Orthogonally Additive Holomorphic functions of Bounded Type over $C(K)$* , Proc. of the Edinburgh Math. Soc. **53** (2010), 609–618.
- [5] S. DINEEN, *Complex Analysis on infinite dimensional Spaces*, Springer, 1999.
- [6] J. A. JARAMILLO, A. PRIETO, I. ZALDUENDO, *Orthogonally additive holomorphic functions on open subsets of $C(K)$* , Rev. Mat. Complut. (2010), Pages 1–11. DOI: 10.1007/s13163-010-0055-2. Article in press.
- [7] H. KOSAKI, *Applications of the complex interpolation method to a von Neumann algebra: noncommutative L^p -spaces*, J. Funct. Anal. **56**, 1 (1984), 29–78.
- [8] C. PALAZUELOS, A. M. PERALTA, AND I. VILLANUEVA, *Orthogonally Additive Polynomials on C^* -Algebras*, Quart. J. Math. **59** (2008), 363–374.
- [9] D. PÉREZ, AND I. VILLANUEVA, *Orthogonally additive polynomials on spaces of continuous functions*, J. Math. Anal. Appl. **306** (2005), 97–105.
- [10] G. PISIER, *Factorization of operators through $L_{p_{\infty}}$ or L_{p_1} and noncommutative generalizations*, Math. Ann. **276**, 1 (1986), 105–136.
- [11] S. SAKAI, *C^* -algebras and W^* -algebras*, Springer-Verlag, Berlin, 1971.
- [12] K. SUNDARESAN, *Geometry of spaces of homogeneous polynomials on Banach lattices, Applied geometry and discrete mathematics*, 571–586, DIMACS Ser. Discrete Math. Theoret. Comput. Sci., 4, Amer. Math. Soc., Prov., RI, 1991.
- [13] M. TAKESAKI, *Theory of operator algebras, I*, Springer-Verlag, New York-Heidelberg, 1979.