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BISEPARATING MAPS BETWEEN SMOOTH VECTOR–VALUED FUNCTIONS ON BANACH MANIFOLDS

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Abstract. An $\mathscr{S}$-category consists all Banach manifolds as objects and subclasses of continuous functions (with some kind of smoothness) as morphisms. This notion covers, for example, the categories C^∞, C^n, C, and Lip_{loc} of all smooth functions, C^n-functions, continuous functions, and local Lipschitz functions. It is shown by Garrido, Jaramillo and Prieto in 2000 that two C^∞-smooth Banach manifolds X and Y are C^∞-diffeomorphic to each other if and only if there is an algebra isomorphism from $C^\infty(X,\mathbb{R})$ onto $C^\infty(Y,\mathbb{R})$. We extend this result to general abstract $\mathscr{S}$-categories, and from algebra isomorphisms of scalar functions to the maps which are linear, bijective and separating, between vector-valued functions.

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