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MONOTONICITY OF GENERALIZED FURUTA TYPE FUNCTIONS

JIANGTAO YUAN AND GUOXING JI

Abstract. The monotonicity of generalized Furuta type operator function $F_{s_0}(r,s) = C^{\frac{-r}{2}}(C^{\frac{r}{2}}(A^{\frac{t}{2}}B^pA^{\frac{t}{2}})^sC^{\frac{-r}{2}})^{\frac{(p+t)s_0+r}{(p+t)s+r}}C^{\frac{-r}{2}}$ is discussed via the equivalent relations between operator inequalities. Let $-1 \leqslant t < 0$, $p \geqslant 1$ $(p+t \neq 0)$, $C \geqslant A \geqslant B \geqslant 0$ with $A > 0$. It is shown that, for each s_0 such that $\frac{t}{p+t} < s_0$, the function $F_{s_0}(r,s)$ is decreasing for both $r \geqslant -t$ and $s \geqslant \max\{1, s_0\}$. Moreover, some examples are given which imply that, for each $s_0 \geqslant 1$ and $r \geqslant -t$, the monotone interval $[s_0, \infty)$ of s in $F_{s_0}(r,s)$ is unique in the interval $[-\frac{r}{p+t}, \infty)$.

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