

ON YUAN–GAO’S “COMPLETE FORM” OF FURUTA INEQUALITY

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Abstract. Recently Yuan and Gao gave a “complete form” of Furuta inequality. We present its extension by an expression of operator mean: If $A \geq B \geq 0$ with $A > 0$, $p \geq p_0 \geq 0$ and $r, r_0 > 0$, then

$$A^{-r_0} \sharp_{\frac{\delta+r_0}{p_0+r_0}} B^{p_0} \geq B^\delta \geq A^{-r} \sharp_{\frac{\delta+r}{p+r}} B^p$$

for $p_0 \leq \delta \leq \min\{p, 2p_0 + \min\{1, r_0\}\}$. Furthermore we also obtain a grand Furuta type inequality related to our extension.

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