

DIFFERENTIATING MATRIX FUNCTIONS

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Abstract. Real-valued functions on \mathbb{R}^d induce matrix-valued functions defined on the space of d -tuples of $n \times n$ pairwise-commuting self-adjoint matrices. We examine the geometry of this space of matrices and conclude that a suitable notation of differentiation of these matrix functions is differentiation along curves. We prove that continuously differentiable real-valued functions induce continuously differentiable matrix functions and give a formula for the derivative. We also show that real-valued m -times continuously differentiable functions defined on open rectangles in \mathbb{R}^2 induce matrix functions that can be m -times continuously differentiated along m -times continuously differentiable curves.

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