HILBERT–SCHAUDER FRAME OPERATORS

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Abstract. We introduce a new concept of frame operators for Banach spaces we call a Hilbert-Schauder frame operator. This is a hybrid between standard frame theory for Hilbert spaces and Schauder frame theory for Banach spaces. Most of our results involve basic structure properties of the Hilbert-Schauder frame operator. Examples of Hilbert-Schauder frames include standard Hilbert frames and classical bases of ℓ_p and L^p-spaces with $1 < p \leqslant 2$. Finally, we give a new isomorphic characterization of Hilbert spaces.

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