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THE PAULSEN PROBLEM IN OPERATOR THEORY

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Abstract. The *Paulsen Problem* in Hilbert space frame theory has proved to be one of the most intractable problems in the field. We will help explain why by showing that this problem is equivalent to a fundamental, deep problem in operator theory. This answers a question posed by Bodmann and Casazza. We will also give generalizations of these problems and we will spell out exactly the *complementary versions* of the problem.

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REFERENCES

[1] M. ARGERAMI AND P. MASSEY, *Towards the Carpenter's theorem*, Proc. Amer. Math. Soc. **137** (2009), 3679–3687.

[2] J. ANTEZANA, P. MASSEY, M. RUIZ, D. STOJANOFF, *The Schur-Horn theorem for operators and frames with prescribed norms and frame operator*, Illinois J. Math. **51** (2007), 537–560.

[3] M. ARGERAMI, P. MASSEY, *A Schur-Horn theorem in* II_1 *factors*, Indiana Univ. Math. J. **56** (2007), 2051–2059.

[4] W. ARVESON, *Diagonals of normal operators with finite spectrum*, Proc. Natl. Acad. Sci. USA **104** (2007), 1152–1158.

[5] W. ARVESON, R. KADISON, *Diagonals of self-adjoint operators*, Operator theory, operator algebras, and applications, 247–263, Contemp. Math., **414**, Amer. Math. Soc., Providence, RI, 2006.

[6] R. BALAN, *Equivalence relations and distances between Hilbert frames*, Proc. Amer. Math. Soc. **127**, 8 (1999), 2353–2366.

[7] B. BODMANN AND P. G. CASAZZA, *The road to equal norm Parseval frames*, Journal of Functional Analysis **258**, 2 (2010), 397–420.

[8] M. BOWNIK, J. JASPER, *Characterization of sequences of frame norms*, J. Reine Angew. Math., to appear.

[9] P. G. CASAZZA, M. FICKUS AND D. MIXON, *Auto-tunning unit norm tight frames*, Appl. and Comp. Harmonic Anal. **32** (2012), 1–15.

[10] P. G. CASAZZA, M. FICKUS, M. LEON, J. KOVAČEVIĆ, J. C. TREMAIN, *A physical interpretation for finite tight frames*, Appl. and Comp. Harmonic Anal. (2006), 51–78.

[11] P. G. CASAZZA AND J. J. KOVAČEVIĆ, *Uniform tight frames with erasures*, Advances in Computational Mathematics **18**, 2–4 (2003), 93–116.

[12] P. G. CASAZZA AND G. KUTYNIOK, *A generalization of Gram-Schmidt orthogonalization generating all Parseval frames*, Advances in Computational math. **18** (2007), 65–78.

[13] P. G. CASAZZA AND M. LEON, *Existence and construction of finite tight frames*, J. Concr. Appl. Math **4**, 3 (2006), 277–289.

[14] P. G. CASAZZA AND M. LEON, *Existence and construction of finite frames with a given frame operator*, International Journal of Pure and Applied Mathematics **63**, 2 (2010), 149–158.

[15] O. CHRISTENSEN, *An Introduction to Frames and Riesz Bases*, Birkhaüser, Boston (2003).

[16] J. H. CONWAY, R. H. HARDIN, N. J. A. SLOANE, *Packing lines, planes, etc.: packings in Grassmannian spaces*, Experiment. Math. **5**, 2 (1996), 139–159.

[17] I. C. GOHBERG AND A. S. MARKUS, *Some relations between eigenvalues and matrix elements of linear operators*, Mat. Sb. (N.S.) **64** (1964), 481–496.

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[18] D. HAN AND D. LARSON, *Frames, Bases and Group Representations*, Mem. Amer. Math Soc. **147** (2000).

[19] A. HORN, *Doubly stochastic matrices and the diagonal of a rotation matrix*, Amer. J. Math. **76** (1954), 620–630.

[20] R. A. HORN AND C. R. JOHNSON, *Matrix analysis*, Cambridge University Press, 1985.

[21] A. J. E. M. JANSSEN, *Zak transforms with few zeroes and the tie*, In *Advances in Gabor Analysis*, H.G. Feichtinger and T. Strohmer (eds.), pages 31–70, Birkhäuser, Boston (2002).

[22] J. JASPER, *The Schur-Horn theorem for operators with three point spectrum*, Preprint.

[23] R. KADISON, *The Pythagorean theorem. I. The finite case*, Proc. Natl. Acad. Sci. USA **99** (2002), 4178–4184.

[24] R. KADISON, *The Pythagorean theorem. II. The infinite discrete case*, Proc. Natl. Acad. Sci. USA **99** (2002), 5217–5222.

[25] K. KORNELSON, D. LARSON, *Rank-one decomposition of operators and construction of frames*, Wavelets, frames and operator theory, 203–214, Contemp. Math., **345**, Amer. Math. Soc., Providence, RI, 2004.

[26] V. KAFTAL AND G. WEISS, *An infinite dimensional Schur-Horn theorem and majorization theory*, J. Funct. Anal., to appear.

[27] A. NEUMANN, *An infinite-dimensional version of the Schur-Horn convexity theorem*, J. Funct. Anal. **161** (1999), 418–451.

[28] I. SCHUR, *Über eine Klasse von Mittelbildungen mit Anwendungen auf die Determinantentheorie*, Sitzungsber. Berl. Math. Ges. **22** (1923), 9–20.

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