

SUCCESSIVE ITERATIONS AND LOGARITHMIC MEANS

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Abstract. The successive iteration (started by Lagrange and Gauss) produces a new mean from two given ones. Several examples of matrix means are given that require the proof of the matrix monotonicity of the corresponding representing function. The paper contains extensions of the logarithmic mean and it is obtained that the Stolarsky mean can be used also for matrices.

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