

NUMERICAL RADIUS AND DISTANCE FROM UNITARY OPERATORS

CATALIN BADEA AND MICHEL CROUZEIX

Abstract. Denote by $w(A)$ the numerical radius of a bounded linear operator A acting on Hilbert space. Suppose that A is invertible and that $w(A) \leq 1 + \varepsilon$ and $w(A^{-1}) \leq 1 + \varepsilon$ for some $\varepsilon \geq 0$. It is shown that $\inf\{\|A - U\| : U \text{ unitary}\} \leq c\varepsilon^{1/4}$ for some constant $c > 0$. This generalizes a result due to J.G. Stampfli, which is obtained for $\varepsilon = 0$. An example is given showing that the exponent $1/4$ is optimal. The more general case of the operator ρ -radius $w_\rho(\cdot)$ is discussed for $1 \leq \rho \leq 2$.

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