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A NEW UPPER BOUND ON THE LARGEST NORMALIZED LAPLACIAN EIGENVALUE

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Abstract. Let $\mathcal{G}$ be a simple undirected connected graph on n vertices. Suppose that the vertices of $\mathcal{G}$ are labelled $1,2,\ldots,n$. Let d_i be the degree of the vertex i. The Randić matrix of $\mathcal{G}$, denoted by R, is the $n\times n$ matrix whose $(i,j)-$entry is $\frac{1}{\sqrt{d_i d_j}}$ if the vertices i and j are adjacent and 0 otherwise. The normalized Laplacian matrix of $\mathcal{G}$ is $\mathcal{L}=I-R$, where I is the $n\times n$ identity matrix. In this paper, by using an upper bound on the maximum modulus of the subdominant Randić eigenvalues of $\mathcal{G}$, we obtain an upper bound on the largest eigenvalue of $\mathcal{L}$. We also obtain an upper bound on the largest modulus of the negative Randić eigenvalues and, from this bound, we improve the previous upper bound on the largest eigenvalue of $\mathcal{L}$.

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Paper OaM-07-19

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