

## UNITAL FULL AMALGAMATED FREE PRODUCTS OF MF $C^*$ -ALGEBRAS

QIHUI LI AND JUNHAO SHEN

*Abstract.* In the paper, we consider the question whether a unital full amalgamated free product of MF algebras is MF. First, we show that, under a natural condition, a unital full free product of two MF algebras with amalgamation over a finite-dimensional  $C^*$ -algebra is again an MF algebra. As an application, we show that a unital full free product of two AF algebras with amalgamation over an AF algebra is an MF algebra if there are faithful tracial states on each of these two AF algebras such that the restrictions on the common subalgebra agree.

*Mathematics subject classification (2010):* 46L09, 46L35.

*Keywords and phrases:* MF  $C^*$ -algebras, unital full amalgamated free products of  $C^*$ -algebras.

### REFERENCES

- [1] S. ARMSTRONG, K. DYKEMA, R. EXEL AND H. LI, *On embeddings of full amalgamated free product  $C^*$ -algebras*, Proc. Amer. Math. Soc. **132** (2004), 2019–2030.
- [2] D. AVITZOUR, *Free Products of  $C^*$ -algebras*, Trans. Amer. Math. Soc. **271** (1982), 423–435.
- [3] B. BLACKADAR AND E. KIRCHBERG, *Generalized inductive limits of finite dimensional  $C^*$ -algebras*, Math. Ann. **307** (1997), 343–380.
- [4] F. BOCA, *A note on full free product  $C$ -algebras, lifting and quasidiagonality*, in Operator Theory, Operator Algebras and Related Topics (Proc. of the 16th Op. Thy. Conference, Timisoara 1996), Theta Foundation, Bucharest, 1997, 51–63.
- [5] L. BROWN, R. DOUGLAS AND P. FILLMORE, *Extensions of  $C^*$ -algebras and K-homology*, Ann. of Math. **105** (1977), 265–324.
- [6] N. P. BROWN AND K. J. DYKEMA, *Popa algebras in free group factors*, J. Reine Angew. Math. **573** (2004), 157–180.
- [7] N. P. BROWN, *On quasidiagonal  $C^*$ -algebras*, Operator algebras and applications, 19–64, Adv. Stud. Pure Math., **38**, Math. Soc. Japan, Tokyo, 2004.
- [8] NATHANIAL P. BROWN AND NARUTAKA OZAWA, *Algebras and Finite-Dimensional Approximations*, American Mathematical Society (March 12, 2008).
- [9] M. CHOI, *The full  $C^*$ -algebra of the free group on two generators*, Pacific J. Math. **87**, 1 (1980), 41–48.
- [10] M.-D. CHOI, *Almost commuting matrices need not be nearly commuting*, Proc. Amer. Math. Soc. **102** (1988), 528–533.
- [11] K. DAVIDSON,  *$C^*$ -algebras by Example*, Amer. Mathematical Society (June 1996).
- [12] BENTON L. DUNCAN, *Exactness of universal free products of finite dimensional  $C^*$ -algebras with amalgamation*, arXiv:0901.2312.
- [13] R. EXEL AND T. LORING, *Finite-dimensional representations of free product  $C$ -algebras*, Internat. J. Math. **3**, 4 (1992), 469–476.
- [14] L. GE AND D. HADWIN, *Ultraproducts of  $C^*$ -algebras*, Operator Theory: Advance and Application, Vol. **127**, 305–326.
- [15] U. HAAGERUP, S. THORBJRNSEN, *A new application of random matrices:  $\text{Ext}(C_{\text{red}}^*(F_2))$  is not a group*, Ann. of Math. (2) **162**, 2 (2005), 711–775.
- [16] D. HADWIN, Q. LI AND J. SHEN, *Topological free entropy dimensions in Nuclear  $C^*$ -algebras and in Full Free Products of Unital  $C^*$ -algebras*, Canadian Journal of Mathematics, **63** (2011), 551–590.

- [17] D. HADWIN AND J. SHEN, *Free orbit dimension of finite von Neumann algebras*, J. Funct. Anal. **249**, 1 (2007), 75–91.
- [18] D. HADWIN AND J. SHEN, *Topological free entropy dimension in unital  $C^*$ -algebras*, J. Funct. Anal., 256 (2009) 2027–2068.
- [19] D. HADWIN AND J. SHEN, *Topological free entropy dimension in Unital  $C^*$ -algebras (II) : Orthogonal Sum of Unital  $C^*$ -algebras*, arXiv:0708.0168v4.,
- [20] D. HADWIN AND J. SHEN, *Some Examples of Blackadar and Kirchberg's MF Algebras*, arXiv:0806.4712v4.
- [21] D. HADWIN AND J. SHEN, *Reduced Free Products of Unital AH Algebras and Blackadar and Kirchberg's MF Algebras*, arXiv:0812.0189v1.
- [22] P. R. HALMOS, *Ten problems in Hilbert space*, Bull. Amer. Math. Soc. **76** (1970), 887–933.
- [23] R. KADISON AND J. RINGROSE, *Fundamentals of the Operator Algebras*, Academic, Orlando, FL, 1983, 1986, Vols. 1 and 2.
- [24] Q. LI, J. SHEN, *On RFD Property of Unital Full Amalgamated Free Products of RFD  $C^*$ -Algebras*, Illinois J. Math, To appear.
- [25] H. LIN, *Almost commuting selfadjoint matrices and applications*, Operator algebras and their applications (Waterloo, ON, 1994/1995) 193–233, Fields Inst. Commun., **13**, Amer. Math. Soc., Providence, RI, 1997.
- [26] D. MCDUFF, *Central sequences and the hyperfinite factor*, Proc. London Math. Soc. (3) **21** (1970), 443–461.
- [27] G. K. PEDERSEN, *Pullback and pushout constructions in  $C^*$ -algebra theory*, J. Funct. Anal. **167** (1999), 243–344.
- [28] D. VOICULESCU, *Around quasidiagonal operators*, Integr. Equ. and Op. Thy. **17** (1993), 137–149.
- [29] D. VOICULESCU, *The topological version of free entropy*, Lett. Math. Phys. **62**, 1 (2002), 71–82.