Operators
and
Matrices
Volume 7, Number 2 (2013), 465–476
doi:10.7153/oam-07-28

NUMERICAL RANGES AND COMPRESSIONS OF S_n–MATRICES

HWA-LONG GAU AND PEI YUAN WU

Abstract. Let A be an n-by-n $(n \geqslant 2)$ S_n-matrix, that is, A is a contraction with eigenvalues in the open unit disc and with $\operatorname{rank}(I_n - A^*A) = 1$, and let $W(A)$ denote its numerical range. We show that (1) if B is a k-by-k $(1 \leqslant k < n)$ compression of A, then $W(B) \subsetneqq W(A)$, (2) if A is in the standard upper-triangular form and B is a k-by-k $(1 \leqslant k < n)$ principal submatrix of A, then $\partial W(B) \cap \partial W(A) = \emptyset$, and (3) the maximum value of k for which there is a k-by-k compression of A with all its diagonal entries in $\partial W(A)$ is equal to 2 if $n = 2$, and $\lceil n/2 \rceil$ if $n \geqslant 3$.

Mathematics subject classification (2010): 15A60.
Keywords and phrases: Numerical range, compression, S_n-matrix, unitary dilation.

REFERENCES

[1] H. BERCOVICI, *Numerical ranges of operators of class C_0*, Linear Multilinear Algebra **50** (2002), 219–222.
[2] I. CHALENDAR, P. GORKIN AND J. R. PARTINGTON, *Numerical ranges of restricted shifts and unitary dilations*, Oper. Matrices **3** (2009), 271–281.
[3] U. DAEPP, P. GORKIN AND R. MORTINI, *Ellipses and finite Blaschke products*, Amer. Math. Monthly **109** (2002), 785–795.
[4] U. DAEPP, P. GORKIN AND K. VOSS, *Poncelet's theorem, Sendov's conjecture, and Blaschke products*, J. Math. Anal. Appl. **365** (2010), 93–102.
[5] M. R. EMBRY, *The numerical range of an operator*, Pacific J. Math. **32** (1970), 647–650.
[6] H.-L. GAU AND P. Y. WU, *Numerical range of $S(\phi)$*, Linear Multilinear Algebra **45** (1998), 49–73.
[7] H.-L. GAU AND P. Y. WU, *Dilation to inflations of $S(\phi)$*, Linear Multilinear Algebra **45** (1998), 109–123.
[8] H.-L. GAU AND P. Y. WU, *Lucas' theorem refined*, Linear Multilinear Algebra **45** (1999), 359–373.
[9] H.-L. GAU AND P. Y. WU, *Numerical range and Poncelet property*, Taiwanese J. Math. **7** (2003), 173–193.
[10] H.-L. GAU AND P. Y. WU, *Numerical range circumscribed by two polygons*, Linear Algebra Appl. **382** (2004), 155–170.
[11] H.-L. GAU AND P. Y. WU, *Numerical range of a normal compression*, Linear Multilinear Algebra **52** (2004), 195–201.
[12] H.-L. GAU AND P. Y. WU, *Numerical range of a normal compression II*, Linear Algebra Appl. **390** (2004), 121–136.
[13] U. HAAGERUP AND P. DE LA HARPE, *The numerical radius of a nilpotent operator on a Hilbert space*, Proc. Amer. Math. Soc. **115** (1992), 371–379.
[14] R. A. HORN AND C. R. JOHNSON, *Topics in Matrix Analysis*, Cambridge Univ. Press, Cambridge, 1991.
[15] B. MIRMAN, *Numerical ranges and Poncelet curves*, Linear Algebra Appl. **281** (1998), 59–85.
[16] B. MIRMAN, *UB-matrices and conditions for Poncelet polygon to be closed*, Linear Algebra Appl. **360** (2003), 123–150.
[17] B. MIRMAN, *Poncelet's porism in the finite real plane*, Linear Multilinear Algebra **57** (2009), 439–458.
[18] B. MIRMAN, V. BOROVIKOV, L. LADYZHENSKY AND R. VINOGRAD, *Numerical ranges, Poncelet curves, invariant measures*, Linear Algebra Appl. **329** (2001), 61–75.

Paper OaM-07-28

[19] B. MIRMAN AND P. SHUKLA, *A characterization of complex plane Poncelet curves*, Linear Algebra Appl. **408** (2005), 86–119.
[20] D. SARASON, *Generalized interpolation in* H^∞, Trans. Amer. Math. Soc. **127** (1967), 179–203.
[21] P. Y. WU, *Polygons and numerical ranges*, Amer. Math. Monthly **107** (2000), 528–540.
[22] T. YOSHINO, *Introduction to Operator Theory*, Longman, Harlow, Essex, 1993.
[23] H.-Y. ZHANG, Y.-N. DOU, M.-F. WANG AND H.-K. DU, *On the boundary of numerical ranges of operators*, Appl. Math. Lett. **24** (2011), 620–622.

Operators and Matrices
www.ele-math.com
oam@ele-math.com