

SATURATION FOR CESÀRO MEANS OF HIGHER ORDER

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Abstract. The classical Cesàro means of higher order are investigated from the point of view of saturation theory. In this direction the Cesàro means for power bounded operators were studied by Butzer-Westphal [1, 2], and certain appropriate results were given by Lin-Sine [6]. In this paper we prove that the behavior of the Cesàro means of higher order concerning the saturation, are essentially of the same form, under the boundedness condition of Cesàro means. Also, some results of Lin-Sine are extended to Cesàro means of higher order.

Mathematics subject classification (2010): Primary 47A10, 47A35; Secondary 47B20.

Keywords and phrases: Cesàro mean, Cesàro bounded operator, ergodicity, saturation.

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