HYPONORMAL TRIGONOMETRIC TOEPLITZ OPERATORS

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Abstract. We investigate hyponormal Toeplitz operators T_ϕ with trigonometric polynomial symbols ϕ via the Carathéodory-Schur Interpolation Problem. We present several formulae for computing the rank of the selfcommutator $[T_\phi^*, T_\phi]$ in the cases where T_ϕ is a hyponormal operator. In addition we consider the hyponormal extension problem of Toeplitz operators.

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