

A POLYNOMIAL IDENTITY AND ITS APPLICATION TO INVERSE SPECTRAL PROBLEMS IN STIELTJES STRINGS

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Abstract. The equation $\Phi = P_1 Q_2 + P_2 Q_1$ is studied where Φ , Q_1 , Q_2 are known real polynomials while P_1 and P_2 are unknown polynomials. Condition are obtained for the solution (P_1, P_2) to exist and to be such that $P_1^{-1} Q_1$ and $P_2^{-1} Q_2$ are Stieltjes functions. This result is used to prove the existence of a tree with two complementary subtrees of Stieltjes strings such that the spectrum of the Neumann boundary value problem on the tree is exactly the set of zeros of Φ and the spectra of Dirichlet problems on the subtrees are the sets of zeros of Q_1 and Q_2 .

This result is generalized to the equation $\Phi = \sum_{i=1}^q \left[P_i \prod_{j \neq i} Q_j \right]$, which is then applied to solve the inverse several spectra problem for trees of Stieltjes strings.

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REFERENCES

- [1] F. V. ATKINSON, *Discrete and Continuous Boundary Problems*, Academic Press, New York, 1964.
- [2] O. BOYKO AND V. PIVOVARCHIK, *Inverse spectral problem for a star graph of Stieltjes strings*, *Methods Funct. Anal. Topology* **14**, 2 (2008), 159–167.
- [3] O. BOYKO AND V. PIVOVARCHIK, *The inverse three-spectral problem for a Stieltjes string and the inverse problem with one-dimensional damping*, *Inverse Problems* **24**, 1 (2008), 015019, 13 pp.
- [4] W. CAUER, *Die Verwirklichung von Wechselstromwiderständen vorgeschriebener Frequenz-abhängigkeit*, *Arch. für Electrotech.* **17**, 4 (1926), 355–388.
- [5] S. COX, M. EMBREE AND J. HOKANSON, *One can hear the composition of a string: experiments with an inverse eigenvalue problem*, *SIAM Review* **54**, 1 (2012), 157–178.
- [6] M. DRIGNEI, *Uniqueness of solutions to inverse Sturm-Liouville problems with $L_2(0, a)$ potentials using three spectra*, *Adv. Appl. Math.* **42**, 4 (2009), 471–482.
- [7] A. DUARTE, *Construction of analytic matrices from spectral data*, *Linear Algebra and Its Applications* **113** (1989), 173–182.
- [8] A. F. FILIMONOV AND A. D. MYSHKIS, *On properties of large wave effect in classical problem bead string vibration*, *J. Difference Equations and Applications* **10**, 13–15 (2004), 1171–1175.
- [9] F. R. GANTMAKHER AND M. G. KREIN, *Oscillating Matrices and Kernels and Vibrations of Mechanical Systems* (Russian), GITTL, Moscow-Leningrad, 1950. German translation: Akademie Verlag, Berlin, 1960.
- [10] F. GESZTESY AND B. SIMON, *On the determination of a potential from three spectra*, in: *Advances in Mathematical Sciences*, V. Buslaev and M. Solomyak, eds., Amer. Math. Soc. Transl. (2) **189** (1999), 85–92.
- [11] R. O. HRYNIV AND YA. V. MYKYTYUK, *Inverse spectral problems for Sturm–Liouville operators with singular potentials. Part III: Reconstruction by three spectra*, *J. Math. Anal. Appl.* **284**, 2 (2003), 626–646.
- [12] I. S. KAC AND M. G. KREIN, *R-functions – analytic functions mapping the upper half-plane into itself*, *Amer. Math. Soc. Translations Ser. 2* **103** (1974), 1–18.

- [13] M. G. KREIN, *On some new problems of the theory of vibrations of Sturm systems*, *Prikladnaya Matematika i Mekhanika* **16**, 5 (1952), 555–568 (Russian).
- [14] P. F. KURCHANOV, A. D. MYSHKIS AND A. M. FILIMONOV, *Train vibrations and Kronecker's theorem*, *Prikladnaya Matematika i Mekhanika* **55**, 6 (1991), 989–995 (Russian).
- [15] C. K. LAW AND V. PIVOVARCHIK, *Characteristic functions of quantum graphs*, *J. Phys. A: Math. Theor.* **42** (2009), 035302 (11p).
- [16] B. M. LEVITAN AND M. G. GASIMOV, *Determination of a differential equation by two of its spectra*, *Uspekhi Mat. Nauk* **19**, 2(116) (1964), 3–63 (Russian).
- [17] B. M. LEVITAN, *Inverse Sturm-Liouville Problems* (Russian), Moscow, Nauka, 1984. English translation: VNU Science Press BV, Utrecht.
- [18] V. MARCHENKO, *Sturm-Liouville Operators and Applications* (Russian), Naukova Dumka, Kiev, 1977. English translation: *Oper. Theory Adv. Appl.* **22**, Birkhäuser Verlag, Basel, 1986.
- [19] P. NYLEN, F. UHLIG, *Realization of interlacing by tree-patterned matrices*, *Linear and Nonlinear Algebra* **38** (1994), 13–37.
- [20] V. PIVOVARCHIK AND H. WORACEK, *Sums of Nevanlinna functions and differential equations on star-shaped graphs*, *Operators and Matrices* **3**, 4 (2009), 451–501.
- [21] V. PIVOVARCHIK, *Inverse problem for the Sturm-Liouville equation on a simple graph*, *SIAM J. Math. Anal.* **32** (2000), 801–819.
- [22] V. PIVOVARCHIK, *Inverse problem for the Sturm-Liouville equation on a star-shaped graph*, *Math. Nachr.* **280** 13–14 (2007), 1595–1619.
- [23] V. PIVOVARCHIK, *Existence of a tree of Stieltjes strings corresponding to two given spectra*, *J. Phys. A: Math. Theor.* **42** (2009), 375213 (16 pp).
- [24] V. PIVOVARCHIK, *An inverse Sturm-Liouville problem by three spectra*, *Integral Equations and Operator Theory* **34**, 2 (1999), 234–243.