

## DETERMINANT INEQUALITIES CONCERNING THE SOLUTION OF WAVE DIFFRACTION PROBLEMS WITH SEVERAL PARALLEL SOMMERFELD HALF PLANES

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*Abstract.* This paper deals with a system of parallel non-staggered half planes as scatterers for plane wave fields incident perpendicularly to the edges. In cases with first or second kind boundary conditions, the determinant of the positive definite real part of the corresponding  $L^2$ -lifted Wiener–Hopf matrix  $H_N(\varepsilon)$ , with  $\varepsilon = \varepsilon_1, \dots, \varepsilon_{N-1}$  to lie in the unit circle, is shown to satisfy an inequality with lower and upper bounds for any positive spacings between the scatterers. The main result is based upon a sharpening of Hadamard’s inequality. The derived relations can be used to prove a priori estimates in the construction of the inverses by operator Neumann series. The matrices which appear in the estimates are suitable to test the computation accuracy by machines via Corollary ?? and are related to infinite products of certain determinants, which could be of interest for number theory.

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