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BLOCK REPRESENTATIONS FOR CLASSES OF ISOMETRIC OPERATORS BETWEEN KREĬN SPACES

RUDI WIETSMA

Abstract. The behavior of isometric and unitary operators between Kreĭn spaces is investigated by means of block decompositions. Therefore two types of isometric operators having a block representation, so-called archetypical isometric operators, are introduced. It is shown that interesting classes of isometric operators, in particular the class of unitary operators, can be expressed as a composition of archetypical isometric operators and bounded unitary operators. As a consequence of these block representations, useful information about the behavior of the isometric operators under consideration can be obtained. In particular, some results on (the Weyl families of) (quasi-) boundary triplets are presented.

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