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FURTHER RESULTS ON GENERALIZED BOTT–DUFFIN INVERSES

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Abstract. Let A be a bounded linear operator, $P_{\mathscr{M}}$ be an orthogonal projection with range $\mathscr{M}$ and $P_{\mathscr{M},\mathscr{N}}$ be an idempotent with range $\mathscr{M}$ and kernel $\mathscr{N}$. This paper presents some novel relations between Bott-Duffin inverse $A^{+}_{\mathscr{M}} = P_{\mathscr{M}}(AP_{\mathscr{M}} + P_{\mathscr{M}^{\perp}})^{+}$ and generalized Bott-Duffin inverse $A^{+}_{\mathscr{M},\mathscr{N}} = P_{\mathscr{M},\mathscr{N}}(AP_{\mathscr{M},\mathscr{N}} + P_{\mathscr{N},\mathscr{M}})^{+}$. Furthermore, the representations for the Bott-Duffin inverse and generalized Bott-Duffin inverse are presented.

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