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SPECTRAL ANALYSIS OF CERTAIN SPHERICALLY HOMOGENEOUS GRAPHS

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Abstract. We study operators on rooted graphs with a certain spherical homogeneity. These graphs are called path commuting and allow for a decomposition of the adjacency matrix and the Laplacian into a direct sum of Jacobi matrices which reflect the structure of the graph. Thus, the spectral properties of the adjacency matrix and the Laplacian can be analyzed by means of the elaborated theory of Jacobi matrices. For some examples which include antitrees, we derive the decomposition explicitly and present a zoo of spectral behavior induced by the geometry of the graph. In particular, these examples show that spectral types are not at all stable under rough isometries.

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