MATRICES WITH DEFECT INDEX ONE

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Abstract. In this paper, we give some characterizations of matrices which have defect index one. Recall that an \( n \)-by-\( n \) matrix \( A \) is said to be of class \( S_n \) (resp., \( S_n^{-1} \)) if its eigenvalues are all in the open unit disc (resp., in the complement of closed unit disc) and rank \((I_n - A^*A) = 1 \). We show that an \( n \)-by-\( n \) matrix \( A \) is of defect index one if and only if \( A \) is unitarily equivalent to \( U \oplus C \), where \( U \) is a \( k \)-by-\( k \) unitary matrix, \( 0 \leq k < n \), and \( C \) is either of class \( S_{n-k} \) or of class \( S_{n-1} \). We also give a complete characterization of polar decompositions, norms and defect indices of powers of \( S_n^{-1} \)-matrices. Finally, we consider the numerical ranges of \( S_n^{-1} \)-matrices and \( S_n \)-matrices, and give a generalization of a result of Chien and Nakazato on tridiagonal matrices (cf. [3, Theorem 7]).


Keywords and phrases: polar decomposition, defect index, numerical range, \( S_n \)-matrix, \( S_n^{-1} \)-matrix.

REFERENCES