

MATRICES WITH DEFECT INDEX ONE

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Abstract. In this paper, we give some characterizations of matrices which have defect index one. Recall that an n -by- n matrix A is said to be of class \mathcal{S}_n (resp., \mathcal{S}_n^{-1}) if its eigenvalues are all in the open unit disc (resp., in the complement of closed unit disc) and $\text{rank}(I_n - A^*A) = 1$. We show that an n -by- n matrix A is of defect index one if and only if A is unitarily equivalent to $U \oplus C$, where U is a k -by- k unitary matrix, $0 \leq k < n$, and C is either of class \mathcal{S}_{n-k} or of class \mathcal{S}_{n-k}^{-1} . We also give a complete characterization of polar decompositions, norms and defect indices of powers of \mathcal{S}_n^{-1} -matrices. Finally, we consider the numerical ranges of \mathcal{S}_n^{-1} -matrices and \mathcal{S}_n -matrices, and give a generalization of a result of Chien and Nakazato on tridiagonal matrices (cf. [3, Theorem 7]).

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