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ONE–DIMENSIONAL SCHRÖDINGER OPERATORS WITH δ'–INTERACTIONS ON A SET OF LEBESGUE MEASURE ZERO

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Abstract. Let Γ be a compact subset of $\mathbb{R}$ of Lebesgue measure zero. The notion 'Schrödinger operator defining a δ'-interaction on Γ' is introduced. The dimension of the range of the spectral projection $\chi_{(-\infty,0)}(A)$ of a Schrödinger operator A defining a δ'-interaction on Γ is not less than the number of isolated points of Γ where the intensity of the δ'-interaction is negative. In the case that the set Γ is endowed with a Radon measure a method how to construct a large class of such operators is presented and for the operators from this class it is shown that their absolutely continuous spectra and their essential spectra are equal to the nonnegative real half-axis. Constructive examples of such operators with infinitely many negative eigenvalues are given.

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