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UNIVERSAL INEQUALITIES FOR EIGENVALUES OF THE LAMÉ SYSTEM

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Abstract. In this paper, we investigate the Dirichlet eigenvalue problem of the Lamé system: $\Delta\mathbf{u}+\alpha\,\mathrm{grad}(\mathrm{div}\mathbf{u})=-\sigma\mathbf{u}$ on a bounded domain Ω in an n-dimensional Euclidean space $\mathbb{R}^n$, where α is a nonnegative constant and $\mathbf{u}$ is a vector-valued function on Ω. We establish a Levitin-Parnovski-type inequality for its eigenvalues, which gives an estimate for the upper bounds of $\sum_{i=1}^{n}\sigma_{i+j}$ for any positive integer j. Moreover, we obtain some other universal inequalities for eigenvalues of this problem.

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