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NORM CONVERGENCE OF SECTORIAL OPERATORS ON VARYING HILBERT SPACES

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Abstract. Convergence of operators acting on a given Hilbert space is an old and well studied topic in operator theory. The idea of introducing a related notion for operators acting on varying spaces is natural. Many previous contributions to this subject consider either concrete examples of perturbations, or an abstract setting where weak or strong convergence of the resolvents is used. However, it seems that the first results on *norm* resolvent convergence in this direction have been obtained only recently, to the best of our knowledge. Here we consider sectorial operators on Hilbert spaces that depend on a parameter. We define a notion of convergence that generalises convergence of the resolvents in operator norm to the case when the operators act on different spaces. In addition, we show that this kind of convergence is compatible with the functional calculus of the operator and moreover implies convergence of the spectrum. Finally, we present examples for which this convergence can be checked, including convergence of coefficients of parabolic problems. Convergence of a manifold (roughly speaking consisting of thin tubes) towards the manifold's skeleton graph plays a prominent role, being our main application.

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