DIAGRAM VECTORS AND TIGHT FRAME
SCALING IN FINITE DIMENSIONS

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Abstract. We consider frames in a finite-dimensional Hilbert space $\mathscr{H}_n$ where frames are exactly the spanning sets of the vector space. The diagram vector of a vector in $\mathbb{R}^2$ was previously defined using polar coordinates and was used to characterize tight frames in $\mathbb{R}^2$ in a geometric fashion. Reformulating the definition of a diagram vector in $\mathbb{R}^2$ we provide a natural extension of this notion to $\mathbb{R}^n$ and $\mathbb{C}^n$. Using the diagram vectors we give a characterization of tight frames in $\mathbb{R}^n$ or $\mathbb{C}^n$. Further we provide a characterization of when a unit-norm frame in $\mathbb{R}^n$ or $\mathbb{C}^n$ can be scaled to a tight frame. This classification allows us to determine all scaling coefficients that make a unit-norm frame into a tight frame.

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