

SIMILARITY OF PERTURBATIONS OF THE SHIFT AND A DIFFERENT PRODUCT OF RATIONAL FUNCTIONS

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Abstract. Necessary and sufficient conditions are given for the similarity between two perturbations of the (backward) shift by rank one operators, under certain assumptions on the perturbations. The proof of similarity is based on an explicit construction of intertwiners between the perturbations. These intertwiners, in turn, are parametrized by the elements of a certain algebra of rational functions, with the group of “circle invertible” elements of this algebra giving rise to invertible intertwiners.

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