

RELATIVE OSCILLATION THEORY FOR JACOBI MATRICES EXTENDED

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Abstract. We present a comprehensive treatment of relative oscillation theory for finite Jacobi matrices. We show that the difference of the number of eigenvalues of two Jacobi matrices in an interval equals the number of weighted sign-changes of the Wronskian of suitable solutions of the two underlying difference equations. Until now only the case of perturbations of the main diagonal was known. We extend the known results to arbitrary perturbations, allow any (half-) open and closed spectral intervals, simplify the proof, and establish the comparison theorem.

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