

C^* -ALGEBRAS GENERATED BY THREE PROJECTIONS

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Abstract. In this short note, we prove that for a C^* -algebra \mathcal{A} generated by n elements, $M_k(\mathcal{A})$ is generated by k mutually unitarily equivalent and almost mutually orthogonal projections for any $k \geq \delta(n) = \min\{k \in \mathbb{N} \mid (k-1)(k-2) \geq 2n\}$. Then combining this result with recent works of Nagisa, Thiel and Winter on the generators of C^* -algebras, we show that for a C^* -algebra \mathcal{A} generated by finite number of elements, there is $d \geq 3$ such that $M_d(\bar{\mathcal{A}})$ is generated by three mutually unitarily equivalent and almost mutually orthogonal projections. Furthermore, for certain separable purely infinite simple unital C^* -algebras and AF -algebras, we give some conditions that make them be generated by three mutually unitarily equivalent and almost mutually orthogonal projections.

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