Operators and Matrices
Volume 8, Number 1 (2014), 139–156
doi:10.7153/oam-08-07

ON DIFFERENT CONCEPTS OF CLOSEDNESS OF LINEAR OPERATORS

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Abstract. The purpose of this paper is to introduce, by means of the extensions of almost closed operators, the notion of almost closable linear operator acting in a Hilbert or Banach space. This class of operators is strictly included in the class of all unbounded linear operators, it contains the set of all closable operators and that of all almost closed operators and is invariant under finite and countable sums, finite products, limits and integrals. We also present some fundamental properties relative to almost closability and we define a locally convex Hausdorff topology in the set of all almost closable operators.

Mathematics subject classification (2010): 47A05, 47B33.

Keywords and phrases: Almost closed extensions, almost closable operators, sums, products, limits, integrals, locally convex Hausdorff topology.

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