

## GENERATORS WITH A CLOSURE RELATION

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*Abstract.* Assume that a block operator of the form  $\begin{pmatrix} A_1 & \\ A_2 & 0 \end{pmatrix}$ , acting on the Banach space  $X_1 \times X_2$ , generates a contraction  $C_0$ -semigroup. We show that the operator  $A_S$  defined by  $A_S x = A_1 \begin{pmatrix} x \\ SA_2 x \end{pmatrix}$  with the natural domain generates a contraction semigroup on  $X_1$ . Here,  $S$  is a boundedly invertible operator for which  $\varepsilon I - S^{-1}$  is dissipative for some  $\varepsilon > 0$ . With this result the existence and uniqueness of solutions of the heat equation can be derived from the wave equation.

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