A FUNCTIONAL ANALYTIC PERSPECTIVE
TO DELAY DIFFERENTIAL EQUATIONS

RAINER PICARD, SASCHA TROSTORFF AND MARCUS WAURICK

Abstract. We generalize the solution theory for a class of delay type differential equations developed in a previous paper, dealing with the Hilbert space case, to a Banach space setting. The key idea is to consider differentiation as an operator with the whole real line as the underlying domain as a means to incorporate pre-history data. We focus our attention on the issue of causality of the differential equations as a characterizing feature of evolutionary problems and discuss various examples. The arguments mainly rely on a variant of the contraction mapping theorem and a few well-known facts from functional analysis.

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