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ANALYTICAL ASPECTS OF ISOSPECTRAL DRUMS

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Abstract. We reexamine the proofs of isospectrality of the counterexample domains to Kac' question 'Can one hear the shape of a drum?' from an analytical viewpoint. We reformulate isospectrality in a more abstract setting as the existence of a similarity transform intertwining two operators associated with elliptic forms, and give several equivalent characterizations of this property as intertwining the forms and form domains, the associated operators and operator domains, and the semigroups they generate. On a representative pair of counterexample domains, we use these criteria to show that the similarity transform intertwines not only the Laplacians with Neumann (or Dirichlet) boundary conditions but also any two appropriately defined elliptic operators on these domains, even if they are not self-adjoint. However, no such transform can intertwine these operators if Robin boundary conditions are imposed instead of Neumann or Dirichlet. We also remark on various operator-theoretic properties of such intertwining similarity transforms.

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