

THE ANDERSON MODEL WITH MISSING SITES

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Abstract. In the present note we show dynamical localization for an Anderson model with missing sites in a discrete setting at the bottom of the spectrum in arbitrary dimension d . In this model, the random potential is defined on a relatively dense subset of \mathbb{Z}^d , not necessarily periodic, i.e., a Delone set in \mathbb{Z}^d . To work in the lower band edge we need no further assumption on the geometric complexity of the Delone set. We use a spatial averaging argument by Bourgain-Kenig to obtain a uniform Wegner estimate and an initial length scale estimate, which yields localization through the Multiscale Analysis for non ergodic models. This argument gives an explicit dependence on the maximal distance parameter of the Delone set for the Wegner estimate. We discuss the case of the upper spectral band edge and the arising need of imposing the (complexity) condition of strict uniform pattern frequency on the Delone set.

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