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A RIEMANNN SURFACE APPROACH FOR DIFFRACTION FROM RATIONAL WEDGES

T. EHRHARDT, A. P. NOLASCO AND F.-O. SPECK

Abstract. This paper aims at the explicit analytical representation of acoustic, electromagnetic or elastic, time-harmonic waves diffracted from wedges in $\mathbb{R}^3$ in a correct setting of Sobolev spaces. Various problems are modelled by Dirichlet or Neumann boundary value problems for the $2D$ Helmholtz equation with complex wave number. They have been analyzed before by several methods such as the Malinzhinets method using Sommerfeld integrals, the method of boundary integral equations from potential theory or Mellin transformation techniques. These approaches lead to results which are particularly useful for asymptotic and numerical treatment. Here we develop new representation formulas of the solutions which are based upon the solutions to Sommerfeld diffraction problems. We make use of symmetry properties, which require a generalization of these formulas to Riemann surfaces in order to cover arbitrary rational angles of the wedge. The approach allows us to prove well-posedness in suitable Sobolev spaces and to obtain explicit solutions in a new, perhaps surprising, form provided the angle is rational, i.e., $\alpha = \pi m/n$ where $m,n \in \mathbb{N}$.

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www.ele-math.com
oam@ele-math.com