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SIMILARITY RESULTS FOR OPERATORS OF CLASS C_0 AND THE ALGEBRA $H^\infty(T)$

RAPHAËL CLOUÂTRE

Abstract. Given two multiplicity-free operators T_1 and T_2 of class C_0 having the same finite Blaschke product as minimal function, the operator algebras $H^\infty(T_1)$ and $H^\infty(T_2)$ are isomorphic and T_1 is similar to T_2. We find conditions under which the norm of the similarity between the operators can be controlled by the norm of the algebra isomorphism. As an application, we improve upon earlier work and obtain results regarding similarity when the minimal function is an infinite product of finite Blaschke products satisfying the generalized Carleson condition.

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