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SELF–ADJOINT BOUNDARY CONDITIONS AND INTERLACING OF EIGENVALUES FOR THE STURM–LIOUVILLE EQUATION ON GRAPHS

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Abstract. Applying the approach of Kostrykin and Schrader, [14], an explicit characterisation of self-adjoint boundary conditions at the nodes or vertices of a graph for the Sturm-Liouville equation is given. This is then proven to be equivalent to the conditions for self-adjointness of the corresponding system boundary value problem with separated boundary conditions. In addition, using an example, it is shown that the complete graph configuration is incorporated in the system boundary condition at the terminal end point. Making use of the separated system formulation, via matrix Prüfer angle techniques, an interlacing property of the eigenvalues for a self-adjoint Sturm-Liouville boundary value problem on a graph is ascertained.

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