# CLASSICAL ADJOINT COMMUTING MAPPINGS ON ALTERNATE MATRICES AND SKEW–HERMITIAN MATRICES

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*Abstract.* Let $n$ be an even integer with $n \geqslant 4$. In this note we study classical adjoint commuting mappings $\psi$ on the space of $n \times n$ alternate matrices, and on the space of $n \times n$ skew-Hermitian matrices with respect to a proper involution, satisfying one of the following conditions:

- $\psi(\mathrm{adj}\,(A + \alpha B)) = \mathrm{adj}\,(\psi(A) + \alpha\psi(B))$
- $\psi(\mathrm{adj}\,(A - B)) = \mathrm{adj}\,(\psi(A) - \psi(B))$ and $\psi$ is surjective

for scalar $\alpha$ and matrices $A, B$ in each respective matrix spaces. Here, $\mathrm{adj}\,A$ denotes the classical adjoint of a matrix $A$.

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