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NEAR INVARIANCE AND SYMMETRIC OPERATORS

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Abstract. Let S be a subspace of $L^2(\mathbb{R})$. We show that the operator M of multiplication by the independent variable has a simple symmetric regular restriction to S with deficiency indices $(1,1)$ if and only if $S = uhK^2_\theta$ is a nearly invariant subspace, with θ a meromorphic inner function vanishing at i. Here u is unimodular, h is an isometric multiplier of $K^2_\theta := H^2 \ominus \theta H^2$ into H^2 and H^2 is the Hardy space of the upper half plane. Our proof uses the dilation theory of completely positive maps.

Mathematics subject classification (2010): 30H10, 46E22, 47B25, 47B32.
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