NEAR INVARIANCE AND SYMMETRIC OPERATORS

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Abstract. Let $S$ be a subspace of $L^2(\mathbb{R})$. We show that the operator $M$ of multiplication by the independent variable has a simple symmetric regular restriction to $S$ with deficiency indices $(1,1)$ if and only if $S = uhK_\theta^2$ is a nearly invariant subspace, with $\theta$ a meromorphic inner function vanishing at $i$. Here $u$ is unimodular, $h$ is an isometric multiplier of $K_\theta^2 := H^2 \ominus \theta H^2$ into $H^2$ and $H^2$ is the Hardy space of the upper half plane. Our proof uses the dilation theory of completely positive maps.


Keywords and phrases: Symmetric operators, Hardy spaces, model subspaces, nearly invariant.

REFERENCES