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APPROXIMATE DOUBLE COMMUTANTS AND DISTANCE FORMULAS

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Abstract. We extend work of the first author concering relative double commutants and approximate double commutants of unital subalgebras of unital C*-algebras, including metric versions involving distance estimates. We prove metric results for AH subalgebras of von Neumann algebras or AF subalgebras of primitive C*-algebras. We prove other general results, including some for nonselfadjoint commutative subalgebras, using C*-algebraic versions of the Stone-Weierstrass and Bishop-Stone-Weierstrass theorems.

Mathematics subject classification (2010): 46L10, 46L54.
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