

BISHOP'S PROPERTY (β), HYPERCYCLICITY AND HYPERINVARIANT SUBSPACES

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Abstract. The question whether every operator on H has an hyperinvariant subspace is one of the most difficult problems in operator theory. The purpose of this paper is to make a beginning on the hyperinvariant subspace problems for another class of operators closely related to the normal operators namely, the class of k -quasi-class A operators. A necessary and sufficient condition for the hypercyclicity of the adjoint of a quasi-class A operator is also presented.

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