

COMMUTATIVITY OF UNBOUNDED NORMAL AND SELF-ADJOINT OPERATORS AND APPLICATIONS

MOHAMMED HICHEM MORTAD

Abstract. Devinatz, Nussbaum and von Neumann established some important results on the strong commutativity of self-adjoint and normal unbounded operators. In this paper, we prove results in the same spirit.

Mathematics subject classification (2010): Primary 47A05; Secondary 47B25.

Keywords and phrases: Unbounded selfadjoint and normal operators, invertible unbounded operators, operator products, strong commutativity.

REFERENCES

- [1] A. ARAI, *Generalized weak Weyl relation and decay of quantum dynamics*, Rev. Math. Phys., **17/9** (2005), 1071–1109.
- [2] S. M. BARNETT, P. M. RADMORE, *Methods in Theoretical Quantum Optics*, Oxford 1997.
- [3] J. B. CONWAY, *A Course in Functional Analysis*, Springer, 1990 (2nd edition).
- [4] A. DEVINATZ, A. E. NUSSBAUM, J. VON NEUMANN, *On the Permutability of Self-adjoint Operators*, Ann. of Math. (2), **62** (1955), 199–203.
- [5] A. DEVINATZ, A. E. NUSSBAUM, *On the Permutability of Normal Operators*, Ann. of Math. (2), **65** (1957), 144–152.
- [6] B. FUGLEDE, *A Commutativity Theorem for Normal Operators*, Proc. Nati. Acad. Sci., **36** (1950), 35–40.
- [7] B. FUGLEDE, *Conditions for Two Selfadjoint Operators to Commute or to Satisfy the Weyl Relation*, Math. Scand., **51/1** (1982), 163–178.
- [8] F. R. GANTMAHER, M. G. KREIN, *O normalnyh operatorah v ermitovom prostranstve* [Russian], Izv. fiz-mat ob-vapri Kazanskom Univ., **1/3** (1929–1930), 71–84.
- [9] F. GESZTESY, J. A. GOLDSTEIN, H. HOLDEN, G. TESCHL, *Abstract Wave Equations and Associated Dirac-Type Operators*, Annali di Matematica. DOI 10.1007/s10231-011-0200-7.
- [10] A. GHEONDEA, *When Are the Products of Normal Operators Normal?* Bull. Math. Soc. Sci. Math. Roumanie (N.S.) **52(100)/2** (2009), 129–150.
- [11] I. GOHBERG, S. GOLDBERG, M. A. KAASHOEK, *Basic Classes of Linear Operators*, Birkhäuser Verlag, Basel, 2003.
- [12] P. E. T. JORGENSEN, *Unbounded Operators: Perturbations and Commutativity Problems*, J. Funct. Anal. **39/3** (1980), 281–307.
- [13] I. KAPLANSKY, *Products of Normal Operators*, Duke Math. J., **20/2** (1953), 257–260.
- [14] M. H. MORTAD, *An Application of the Putnam-Fuglede Theorem to Normal Products of Selfadjoint Operators*, Proc. Amer. Math. Soc. **131** (2003), 3135–3141.
- [15] M. H. MORTAD, *On Some Product of Two Unbounded Self-adjoint Operators*, Integral Equations Operator Theory, **64** (2009), 399–408.
- [16] M. H. MORTAD, *On the Closedness, the Self-adjointness and the Normality of the Product of Two Unbounded Operators*, Demonstratio Math., **45/1** (2012), 161–167.
- [17] M. H. MORTAD, *On the Normality of the Sum of Two Normal Operators*, Complex Anal. Oper. Theory, **6/1** (2012), 105–112. DOI: 10.1007/s11785-010-0072-7.
- [18] M. H. MORTAD, *On the Normality of the Unbounded Product of Two Normal Operators*, Concrete Operators, **1** (2012), 11–18. DOI: 10.2478/conop-2012-0002.

- [19] E. NELSON, *Analytic Vectors*, Ann. of Math. (2), **70** (1959), 572–615.
- [20] A. E. NUSSBAUM, *A Commutativity Theorem for Unbounded Operators in Hilbert Space*, Trans. Amer. Math. Soc. **140** (1969), 485–491.
- [21] A. E. NUSSBAUM, *A Commutativity Theorem for Semibounded Operators in Hilbert Space*, Proc. Amer. Math. Soc. **125/12** (1997), 3541–3545.
- [22] C. R. PUTNAM, *On Normal Operators in Hilbert Space*, Amer. J. Math., **73** (1951), 357–362.
- [23] M. REED, B. SIMON, *Functional Analysis, Methods of Modern Mathematical Physics*, Vol. **1**, Academic Press, 1972.
- [24] W. RUDIN, *Functional Analysis*, McGraw-Hill, 1991 (2nd edition).
- [25] K. SCHMÜDGEN, J. FRIEDRICH, *On Commuting Unbounded Selfadjoint Operators II*, Integral Equations Operator Theory, **7/6** (1984), 815–867.
- [26] K. SCHMÜDGEN, *Strongly Commuting Unbounded Selfadjoint Operators and Commutants of Unbounded Operator Algebra*, Proc. Amer. Math. Soc. **102/2** (1988), 365–372.
- [27] K. SCHMÜDGEN, *Unbounded Self-adjoint Operators on Hilbert Space*, Springer GTM **265**, (2012).
- [28] Z. SEBESTYÉN, J. STOCHEL, *On Products of Unbounded Operators*, Acta Math. Hungar., **100/1-2** (2003), 105–129.
- [29] T. TUDOR, A. GHEONDEA, *Pauli Algebraic Forms of Normal and Non-Normal Operators*, J. Opt. Soc. Am. A, **24** (2007), 204–210.
- [30] J. WEIDMANN, *Linear operators in Hilbert spaces* (translated from the German by J. Szücs), Springer-Verlag, GTM **68** (1980).
- [31] N. A. WIEGMANN, *Normal Products of Matrices*, Duke Math J., **15**, (1948), 633–638.
- [32] N. A. WIEGMANN, *A Note on Infinite Normal Matrices*, Duke Math. J., **16** (1949), 535–538.
- [33] S. M. ZAGORODNYUK, *On commuting Symmetric Operators*, Methods of Functional Analysis and Topology, 18/2 (2012), 198–200.