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ON CERTAIN FUNCTIONAL EQUATION RELATED TO A CLASS OF GENERALIZED INNER DERIVATIONS

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Abstract. The main purpose of this paper is to prove the following result. Let X be a real or complex Banach space, let $\mathscr{L}(X)$ be the algebra of all bounded linear operators on X and let $\mathscr{A}(X) \subseteq \mathscr{L}(X)$ be a standard operator algebra. Suppose there exists an additive mapping $T : \mathscr{A}(X) \to \mathscr{L}(X)$ satisfying the relation $T(A^n) = T(A)A^{n-1} - AT(A^{n-2})A + A^{n-1}T(A)$ for all $A \in \mathscr{A}(X)$ and some fixed integer $n > 2$. In this case T is of the form $T(A) = AB + BA$ for all $A \in \mathscr{A}(X)$ and some fixed $B \in \mathscr{L}(X)$.

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Paper OaM-08-35

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