SELF–CONJUGATE DIFFERENTIAL AND DIFFERENCE OPERATORS ARISING IN THE OPTIMAL CONTROL OF DESCRIPTOR SYSTEMS

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Abstract. We analyze the structure of the linear differential and difference operators associated with the necessary optimality conditions of optimal control problems for descriptor systems in continuous- and discrete-time. It has been shown in [27] that in continuous-time the associated optimality system is a self-conjugate operator associated with a self-adjoint pair of coefficient matrices and we show that the same is true in the discrete-time setting. We also extend these results to the case of higher order systems. Finally, we discuss how to turn higher order systems with this structure into first order systems with the same structure.

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