

STAR PARTIAL ORDER–HEREDITARY SUBSPACES IN $\mathcal{B}(\mathcal{H})$

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Abstract. Let $\mathcal{B}(\mathcal{H})$ be the algebra of all bounded linear operators on a complex Hilbert space \mathcal{H} . It is proved that a weak operator topology closed nonzero subspace \mathcal{M} in $\mathcal{B}(\mathcal{H})$ is hereditary with respect to the star partial order, that is, for any $A \in \mathcal{B}(\mathcal{H})$ and $B \in \mathcal{M}$, $A \in \mathcal{M}$ whenever $A^*A = A^*B$ and $AA^* = BA^*$, if and only if there is a unique pair of nonzero projections P and Q in $\mathcal{B}(\mathcal{H})$ such that $\mathcal{M} = P\mathcal{B}(\mathcal{H})Q$.

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