

RANK-ONE PERTURBATIONS OF NORMAL OPERATORS AND HYPONORMALITY

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Abstract. Let $T = N + u \otimes v$ be a rank-one perturbation of a normal operator N acting on a separable, infinite dimensional, complex Hilbert space \mathcal{H} . It is proved that the hyponormality of T is equivalent to the normality of T . Some characterizations of hyponormality[normality] of T are obtained.

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