

LINEAR TRANSFORMATIONS WITH CHARACTERISTIC SUBSPACES THAT ARE NOT HYPERINVARIANT

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Abstract. If f is an endomorphism of a finite dimensional vector space over a field K then an invariant subspace $X \subseteq V$ is called hyperinvariant (respectively, characteristic) if X is invariant under all endomorphisms (respectively, automorphisms) that commute with f . According to Shoda (Math. Zeit. 31, 611–624, 1930) only if $|K| = 2$ then there exist endomorphisms f with invariant subspaces that are characteristic but not hyperinvariant. In this paper we obtain a description of the set of all characteristic non-hyperinvariant subspaces for nilpotent maps f with exactly two unrepeated elementary divisors.

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