

STRONG COMMUTATIVITY PRESERVING GENERALIZED DERIVATIONS ON TRIANGULAR RINGS

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Abstract. Let $\mathcal{U} = \text{Tri}(A, M, B)$ be a triangular ring such that either A or B has no nonzero central ideals. It is shown that every pair of strong commutativity preserving generalized derivations g_1, g_2 of \mathcal{U} (i.e., $[g_1(x), g_2(y)] = [x, y]$ for all $x, y \in \mathcal{U}$) is of the form $g_1(x) = \lambda^{-1}x + [x, u]$ and $g_2(x) = \lambda^2 g_1(x)$, where $\lambda \in Z(\mathcal{U})$, the center of \mathcal{U} , and $u \in \mathcal{U}$ with $u[\mathcal{U}, \mathcal{U}] = 0 = [\mathcal{U}, \mathcal{U}]u$. As consequences, every pair of strong commutativity preserving generalized derivations on upper triangular matrix rings and nest algebras is determined.

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