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LOCALLY QUASI–NILPOTENT ELEMENTARY OPERATORS

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Abstract. Let A be a unital dense algebra of linear mappings on a complex vector space X. Let $\phi = \sum_{i=1}^{n} M_{a_i,b_i}$ be a locally quasi-nilpotent elementary operator of length n on A. We show that, if $\{a_1,\dots,a_n\}$ is locally linearly independent, then the local dimension of $V(\phi) = \operatorname{span}\{b_i a_j : 1 \leqslant i,j \leqslant n\}$ is at most $\frac{n(n-1)}{2}$. If $\operatorname{ldim} V(\phi) = \frac{n(n-1)}{2}$, then there exists a representation of ϕ as $\phi = \sum_{i=1}^{n} M_{u_i,v_i}$ with $v_i u_j = 0$ for $i \geqslant j$. Moreover, we give a complete characterization of locally quasi-nilpotent elementary operators of length 3.

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